



Short Communication

Note on “approximation space on novel granulations”

A. Kandil^a, Amr Zakaria^{b,*}^a Department of Mathematics, Faculty of Science, Helwan University, Egypt^b Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt

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ABSTRACT

In this note, we show that an alleged properties stated in [1] are invalid in general, by giving a counter-examples.

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1. Introduction

Definition 1 [2]. Let U be a nonempty set and R be a binary relation on U . The *minimal neighbourhood* of $x \in U$ is defined as:

$$\langle x \rangle_R = \cap \{pR : x \in pR\}, \quad (1)$$

where $pR = \{q \in U : (p, q) \in R\}$.

Proposition 4 of [1] asserted that for any reflexive binary relation on a nonempty set U and $X \subseteq U$ the following pair of *lower and upper approximations*

$$\underline{apr}(X) = \bigcup_{\langle x \rangle_R \subseteq X} \langle x \rangle_R, \quad (2)$$

$$\overline{apr}(X) = \bigcap \{U - \langle x \rangle_R : X \subseteq U - \langle x \rangle_R\}. \quad (3)$$

satisfy the following properties:

- (L₁) $\underline{apr}(X) = [\overline{apr}(X^c)]^c$,
- (U₁) $\overline{apr}(X) = [\underline{apr}(X^c)]^c$,
- (U₃) $\overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y)$,
- (U₄) $\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y)$,
- (U₅) $X \subseteq Y \Rightarrow \overline{apr}(X) \subseteq \overline{apr}(Y)$,
- (U₆) $\overline{apr}(U) = U$,
- (U₇) $X \subseteq \overline{apr}(X)$.

These properties are wrong in general. Moreover, counter-examples are mentioned to prove our claim.

2. Counter-example

The following example shows that (L_1) , (U_1) , (U_3) , (U_4) , (U_5) , (U_6) and (U_7) of [Proposition 4, 1] are wrong, in general.

Example 2. Let $U = \{a, b, c, d\}$ and $R = \Delta \cup \{(a, b), (a, d), (b, d), (c, d), (d, b)\}$, where Δ is the identity relation on U . Hence $\langle a \rangle_R = \{a, b, d\}$, $\langle b \rangle_R = \{b, d\}$, $\langle c \rangle_R = \{c, d\}$ and $\langle d \rangle_R = \{d\}$.

- (L₁) Let $X = \{a, b\}$. Then $[\overline{apr}(X^c)]^c = [\overline{apr}(\{c, d\})]^c = \emptyset^c = U$ and $\underline{apr}(\{a, b\}) = \emptyset$. Hence $\underline{apr}(X) = [\overline{apr}(X^c)]^c$ is incorrect.
- (U₁) Let $X = U$. Then $[\underline{apr}(X^c)]^c = U$, but $\overline{apr}(X) = \emptyset$. Thus $\overline{apr}(X) \neq [\underline{apr}(X^c)]^c$.
- (U₃) Let $X = \{d\}$ and $Y = \{a, c\}$. This implies that $\overline{apr}(X) = \emptyset$, $\overline{apr}(Y) = \{a, c\}$. Hence $\overline{apr}(X \cup Y) = \emptyset$ and $\overline{apr}(X) \cup \overline{apr}(Y) = \{a, c\}$. As a result, $\overline{apr}(X \cup Y) \neq \overline{apr}(X) \cup \overline{apr}(Y)$.
- (U₄) Let $X = \{c\}$ and $Y = \{c, d\}$, then $\overline{apr}(X) = \{c\}$, $\overline{apr}(Y) = \emptyset$ and $\overline{apr}(X \cap Y) = \{c\}$. Thus $\overline{apr}(X \cap Y) \not\subseteq \overline{apr}(X) \cap \overline{apr}(Y)$.
- (U₅) Let $X = \{a\}$ and $Y = \{a, d\}$. Then $\overline{apr}(X) = \{a\}$ and $\overline{apr}(Y) = \emptyset$. Hence $X \subseteq Y$, but $\overline{apr}(X) \not\subseteq \overline{apr}(Y)$.
- (U₆) $\overline{apr}(U) = \emptyset$. Therefore, $\overline{apr}(U) = U$ is invalid.
- (U₇) Let $X = \{b, d\}$. Then $\overline{apr}(X) = \emptyset$. Thus $X \not\subseteq \overline{apr}(X)$. Consequently, $X \subseteq \overline{apr}(X)$ is wrong.

* Corresponding author.

E-mail address: amr.zakaria@edu.asu.edu.eg (A. Zakaria).

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